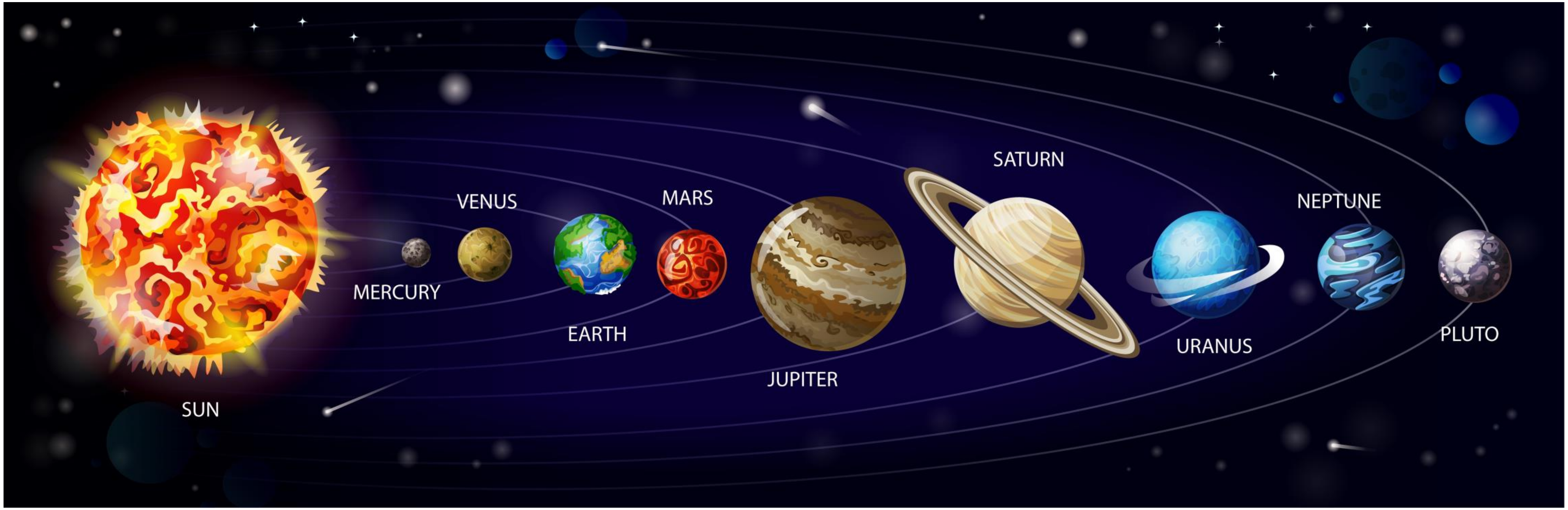


ROTATIONAL KINEMATICS, CENTRIPETAL FORCE AND GRAVITATION

NJ-OER TOPIC-6

Learning Outcomes

- Define arc length, angular displacement, radius of curvature, angular velocity and angular acceleration
- Apply kinematics equations for rotational motion
- Establish the expression for centripetal acceleration.
- Identify the central force that causes centripetal acceleration
- Apply rolling condition to acceleration velocity and displacement
- Explain gravitational force between planets, stars and sattellites
- Calculate escape velocity
- Use Keppler's third law to calculate period or distance from a star for a planet.

Concepts

θi= final angular displacement
θf= final angular displacement
ωi= initial angular velocity
ωf= initial angular velocity
α= angular acceleration
ωavg= average angular velocity
αavg = average angular acceleration
t = time
Δ ω= change in angular velocity
Δ θ= angular displacement
Δ t =change in time
s = distance travelled/arclength
r = radius

vT= tangential velocity
aT= tangential acceleration
ac = centripetal acceleration
F_c = centripetal force
Fn= Normal Force
Ft= Force of tension
f= friction
g= gravitational acceleration
G = Universal gravitational constant
m = mass
vesc= Escape velocity
T = Time Period of a planet
K_s = Kepler's constant

Units

SI UNITS
Position, displacement, radius are in meters “m”
Velocity and speed are in meters per second “m/s”
Acceleration is in meters per second square “m/s^2”
angular displacement is in “radians”
angular velocity is in “ rad/s”
angular acceleration in “rad/s^2”
Force in Newton’s”N”
time and period are in seconds
mass is in kg

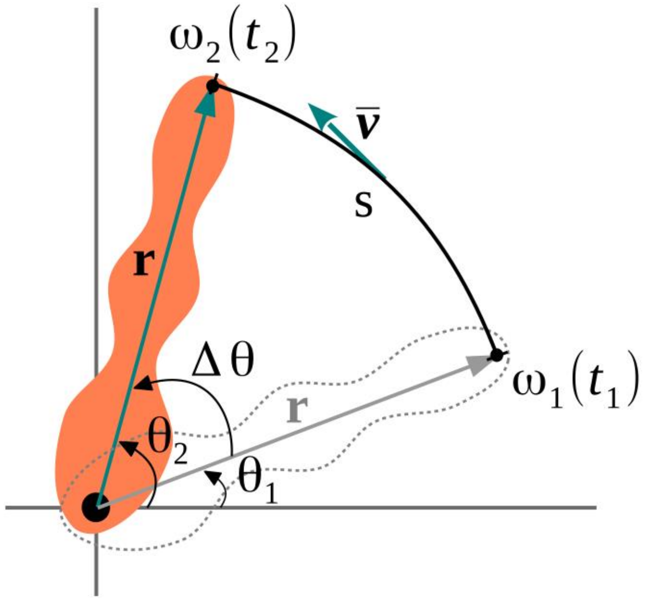
Formulas and Constants

$\Delta\theta=(\omega f+\omega i)t/2$
$\omega avg=(\omega f+\omega i)/2$
$\Delta\theta=\omega i\, t + ½\, \alpha t^2$
$\omega f=\omega i+\alpha t$
$\omega f^2=\omega i^2+2\alpha\Delta\theta$
As a convention, for $\omega f, \omega i, \alpha$ and $\Delta\theta$
clockwise is negative
counter clockwise is positive
$\omega f=\omega$ and $\omega i=\omega o$ when $ti=0$
$s=r\Delta\theta$
$vT=r\omega$
$aT=r\alpha$
$a_{c\,=}\, v^2/r$
$a_c = r\omega^2$
$a^2 = (a_c)^2 + (a_t)^2$
$\mathbf{F_c} = \Sigma\, \mathbf{F}$ use only central and radial forces
$F_c = m\, v^2/r$
$F_c = m\, \omega^2 r$

$g=GM/R^2$
$v^2=GM/R$
$P.E=-GMm/R$
$T^2= 4\pi^2 GMR^{3/2}$
$T^2 = Ks\, r^3$
$T_1{}^2 / r_1{}^3 = T_2{}^2/r_2{}^3$
$G = 6.673\ 10^{-11}\ N\ m^2\ /kg^2$

ROTATIONAL KINEMATICS IMPORTANT TABLES

Rotational	Translational	
$\Delta\theta=\omega avg\,t$	$\Delta x=vavg\,t$	
$\omega=\omega_0+\alpha t$	$v=v_0+at$	constant α
$\Delta\theta=\omega_0 t+½\,\alpha t^2$	$\Delta x=v_0 t+½\,at^2$	constant α
$\omega^2=\omega o^2+2\alpha\Delta\theta$	$v^2=vo^2+2a\Delta x$	constant α



Degree Measures	Radian Measure
30^o	$\pi/6$
60^o	$\pi/3$
90^o	$\pi/2$
120^o	$2\pi/3$
135^o	$3\pi/4$
180^o	π

As a convention, for ωf (ω), ωi (ωo), α and $\Delta\theta$ clockwise is negative and counter clockwise is positive.
They should be treated as vectors

KEY STRATEGIES

Draw the motion diagram
Extract values from the word problem
Identify the unknowns
Find the right starting equation
Plug in the values and do the algebra

At rest means $\omega i=0$
Stops means $\omega f=0$
Constant angular velocity means $\alpha=0$

There are 5 variables and 3 equations.
Each equation is missing one variable.

$\Delta\theta=\omega i\ t + \frac{1}{2}\ \alpha t^2$
This equation can't be used for finding ωf

$\omega f=\omega i+\alpha t$
This equation can't be used for finding $\Delta\theta$

$\omega f^2=\omega i^2+2a\Delta\theta$
This equation can't be used for finding t

CLASSWORK ON ROTATIONAL KINEMATICS

Q1) A drill has two modes. At the faster mode it turns with 30 radians/s and at the slower mode it moves with 20 radians/s. What should be the acceleration so that it would change from the faster mode to the slower mode in 50 turns. (1 Turn is 2pi in radians)

Q2) A wheel, rotating initially at an angular speed of 0.50 rad/s, accelerates over a 7.0-s interval at a rate of 0.040 rad/s^2. What is its angular speed after this 7.0-s interval? What is the angular displacement?

Q3) A wheel, rotating initially at an angular speed of 0.50 rad/s, decelerated over a 7.0-s interval at a rate of 0.040 rad/s^2. What is its angular speed after this 7.0-s interval? What is the angular displacement?

Q4) A disc rotating clockwise with angular speed 4.2 rad/s reverses its direction to rotate counterclockwise with 4.2 rad/s. What is the angular acceleration if change of direction happens in 4.0 seconds? What is the average angular velocity for this motion?

Q5) A hard disc at rest goes to its operational mode of 7200 RPM in 2.4 seconds. What is its operational angular velocity in rad/s and what should be its angular acceleration. 1RPM = (2π radians)/(60 seconds)

$\Delta\theta = \omega i\, t + \frac{1}{2}\alpha t^2$ $\omega f = \omega i + \alpha t$ $\omega f^2 = \omega i^2 + 2\alpha\, \Delta\theta$

CLASSWORK ON ROLLING

$s=r\Delta\theta$
arc length, distance
$v_T=r\omega$
tangential speed

$a_T=r\alpha$
tangential acceleration

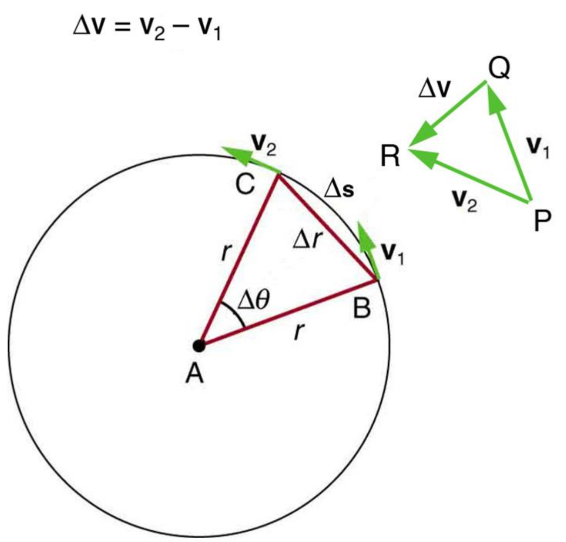
Q) An automobile tire with radius 0.50 meters initially rolling with an angular velocity of 4 rad/s start to accelerate with a rate of $a = 2.3\ m/s^2$.

a)What is the linear speed at the center of the tire and at the bottom of the tire in the beginning?

b)What is the angular acceleration

c) What is the final angular velocity and angular displacement after 3.2 seconds.

CENTRIPETAL FORCE PROBLEMS FIND THE MISSING FORCE



$a_c = v^2/r$ Apply Newton's second law

$F_c = m\, v^2/r$

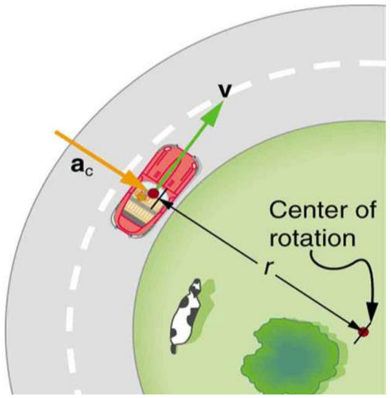
Force equation could be misleading

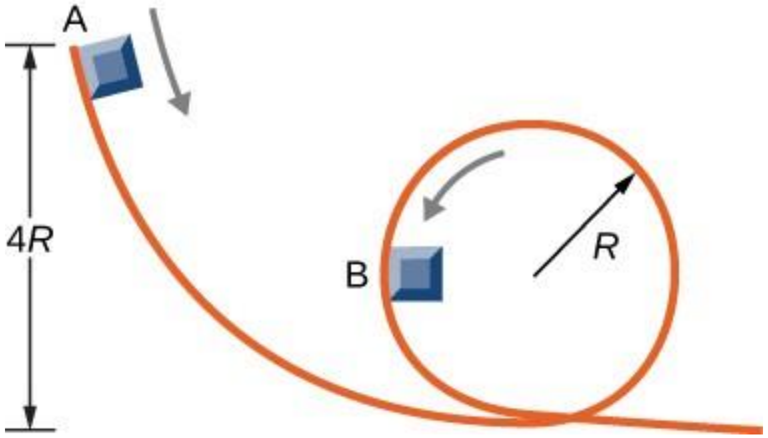
For complex problems, you will not get Fc from $F_c = m\, v^2/r$
You should get Fc from the free body diagram

RULES
1) Forces towards the center are positive
2) Forces away from the center such as radially outward forces are negative
3) Tangential forces which are perpendicular to the radial direction makes no contribution
4) For diagonal forces, take the component towards the center

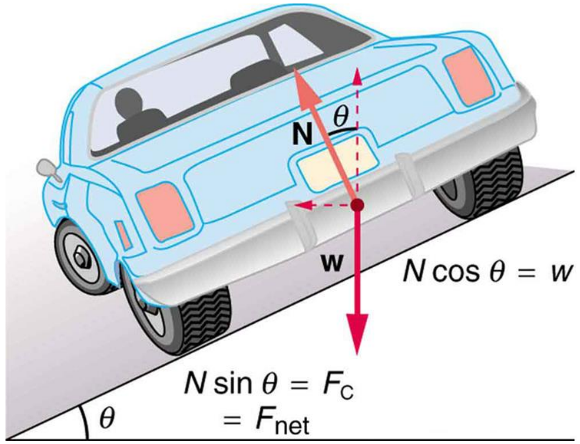
CENTRIPETAL FORCE

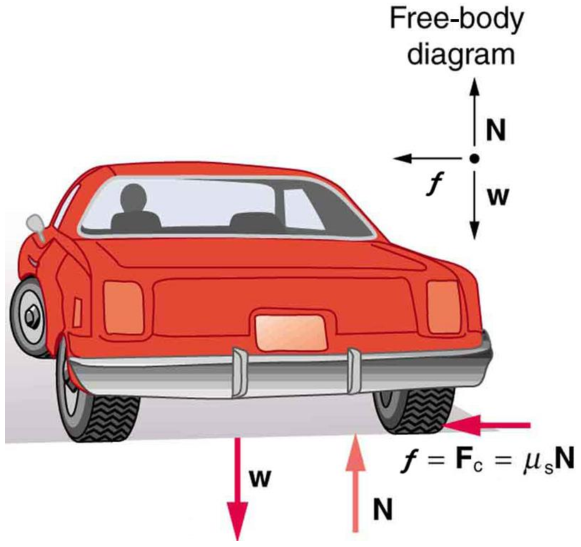
Banked curve, conical pendulum has diagonal forces. Use central component



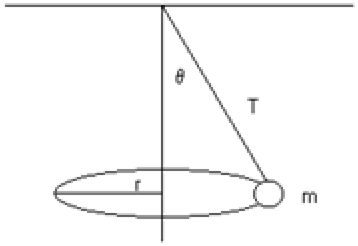


Roller coaster or sliding mass problem. Normal force depends on the position on the circular track. Object must be released from 4R to complete the cycle.





A car taking a curve
Fc= friction



Conical Pendulum
Fc=T sinθ

CLASSWORK CENTRIPETAL FORCE

Q1) A 80 kg passengers in a roller coaster car is moving in a circular loop of radius 10 m, which makes its passengers go upside down at the top.
a) What is the normal force exerted by the tracks if its velocity is 12 m/s at the top?
b) What is the normal force exerted by the tracks if its velocity is 18 m/s at the bottom?

Q2) A conical pendulum consists of a mass of 0.5 kg attached at one end of a string of length 1.0 m. The other end is fixed. As the mass moves in a circular path of radius 0.8 m, the string traces out the surface of a cone. What is the Tension on the string? What is the velocity? (Find the angle or sin(theta), cos(theta) first)

Q3) A 70 kg jet pilot is in a circular dive with the speed of 200 m/s moving in a radius of 4000 meters. What is the normal force that the seat is exerting on a pilot at the bottom of the circular dive?

Q4)A 0.60-kg rock is swung in a circular path and in a vertical plane on a 0.50-m-length string. At the top of the path, its speed is 4 m/s. What is the tension in the string at that point?

Q5) A 0.50-kg toy is rotated in a circular path of radius 1.5 meters on a horizontal table. What is the friction force when it has an angular speed of 2.0 rad/s? What is the minimum coefficient of friction so that it doesn't slide.

CLASSWORK GRAVITATION

1) Gravitational constant on the moon is 1/6 of the Earth's gravity. Earth is 12 times more massive than moon. What is the ratio of radius of the moon to the radius of Earth? What is the escape velocity from the Moon.

2) A satellite moves in a circular orbit around earth at a speed of 5000 m / s
(a)Determine the distance of the satellite from the center of the Earth
(b)Determine the period of the satellites orbit.
(c) If the mass of the satellite is doubled, will the orbital speed increase, decrease of stay the same.

3) Calculate the speed and the distance from the center of the Earth for a geosynchronous satellite

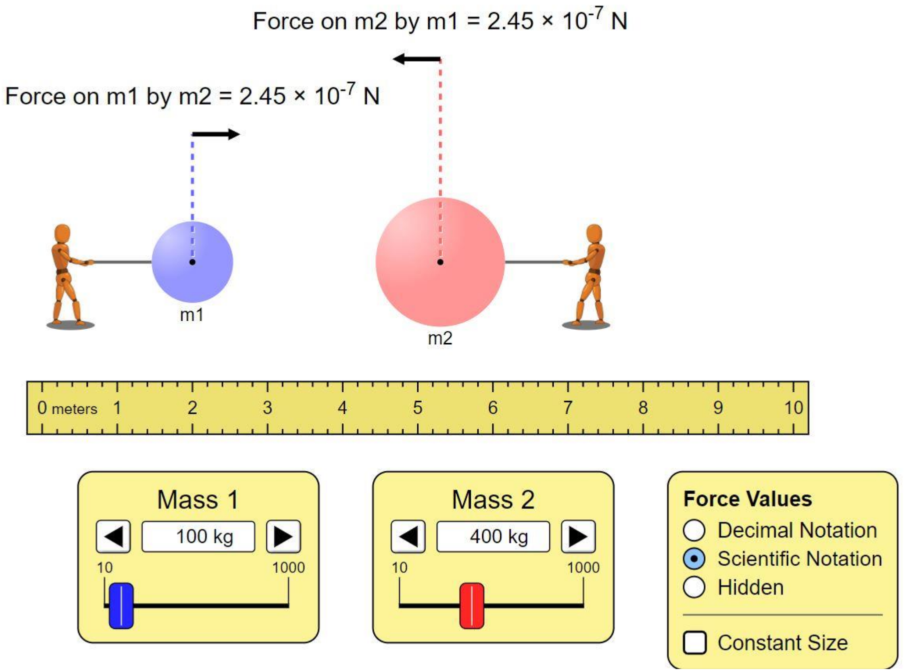
4) The planet Mars has a satellite, Phobos, which travels in an orbit of radius 9.4×10^6m with a period of 7 hr 39 min. Calculate the mass of Mars from this information.

5) A planet X has a period around a star given as 27 years. Another planet Y is 8 times further away from the star. What is the period of that planet Y?

$Ks = 4\pi^2/(G M)$ $T^2 = Ks\, r^3$ $T_1^{\ 2} / r_1^{\ 3} = T_2^{\ 2} / r_2^{\ 3}$ $G = 6.673\ 10^{-11}\ N\, m^2 / kg^2$
$g = GM/R^2$ $g1\, R1^2 / M1 = g2\, R2^2 / M2$ $vesc^2 = 2GM/R$

ACTIVITY GRAVITATIONAL FORCE

Open https://phet.colorado.edu/sims/html/gravity-force-lab/latest/gravity-force-lab_en.html
Adjust Masses and the distance.
Read the force from the app and compare with what you calculated



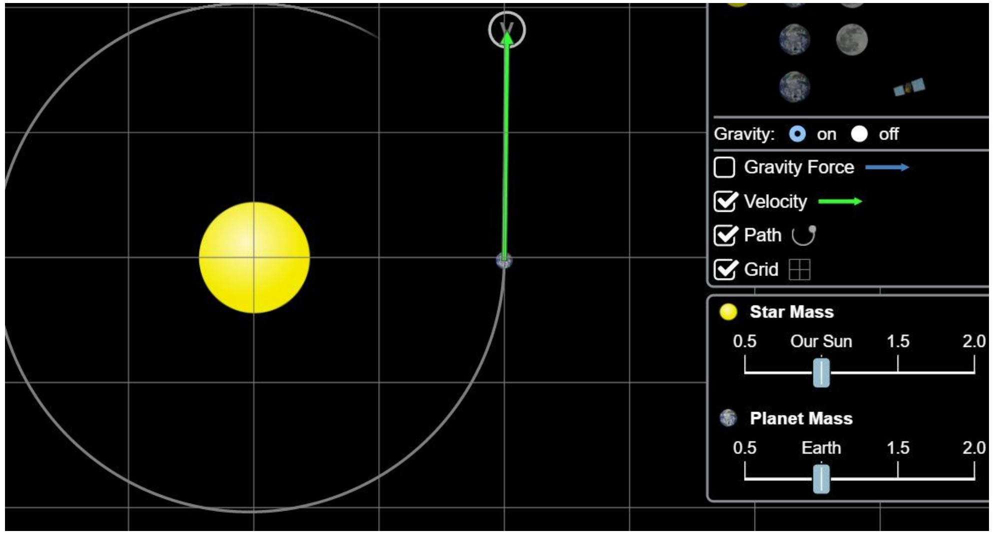
The distance between the mass M1 and M2 is R.
Use scientific notation
$G=6.67\ 10^{-11}\ Nm^2/kg^2$
$F(calculated) = G\ M1\ M2/R^2$

M1(kg)	M2(kg)	R(m)	F(calculated) "N"	F(measured) "N"
100	400	4		
400	400	4		
400	400	2		
200	500	8		

ADVANCED ACTIVITY KEPPLER'S LAW

Open https://phet.colorado.edu/en/simulation/gravity-and-orbits
Click on scale mode, from the options choose grids and velocity and path
Start the app, record the time for one full revolution.
The first trial will correspond to R1=2grids=1 astronomical unit (a.u) v=1.8 unit grids and the period is 1 year
1 a.u. = 1.5 10^8 km, one grid is 7.5 10^7 km. Theoretical Keppler's constant in this unit system is 1 Ks=1.
Change the starting distance R1 and the velocity based on the table below.
Record the distances for semi-major axis R1,R2 convert to a.u. Take the average and find the average distance R
Calculate the Keppler constant for each case using T^2/R^3 = Ks R is in a.u T is in years. Ks calculated should be close to 1.

R1(grids)	R1(a.u)	R2(a.u)	R	v	T(days)	T(years)	Ks
2	1	1	1	1.8	365	1	1
2.5				1.2			
3				1			
1				2.5			



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- Slide 6: Adobe id= 441481751 Kinematic variables of rotational motion: Angular Displacement, Angular Velocity and Angular Acceleration By ScientificStock
- Side 10-11: Open Stax College Physics online textbook
- Slide 14-15 Screenshot from PhET Interactive Simulations University of Colorado Boulder https://phet.colorado.edu